TRANSIT TIMING VARIATIONS
REVISITING KEPLER-36

REÑÉ TRONSGAARD RASMUSSEN
CAMILLA AGENTOFT
MARIA HJORTH

SUPERVISOR: CAROLINA VON ESSEN

AARHUS UNIVERSITY
November 11, 2014
1 Abstract

In this report the system of Kepler-36 has been examined. The system consists of two known planets in approximately orbital resonance with each other; one rocky super-Earth (Kepler-36b) and one Neptune-sized (Kepler-36c). The system was examined because it shows variation in the transit times, or TTVs. Using data from the Kepler satellite, the TTVs were detected for Kepler-36c. This was not possible for Kepler-36b because the transits were not deep enough to be detected using our analysis.

We compared our results with Carter et al. 2012, and found a new average period of Kepler-36c. Since the transits of Kepler-36b could not be detected, the maximum transit timing deviation was only found for Kepler-36c. Therefore the individual masses of the planets could not be calculated, but it was possible to find the relation between them. This fitted well with the data from the article.

If we were to use the RV-method here, we would only get signals in the order of $K = 1 - 2 \text{ m/s}$, which is a very faint signal. The strength of the TTV method is that it is possible to determine the mass of the planets in a multiplanetary system, which gives more information than regular transit photometry.

2 Introduction

Since the beginning of the Kepler-mission the number of known exoplanets has sky-rocketed. Some of the planetary systems consist of a single planet, but many systems have multiple planets. The planets in these systems are therefore not only gravitationally affected by their host stars, but also by each other. This leads to a non-constant transit period, which makes it harder to detect the planets. But it also gives the opportunity to determine the mass of the planets, which is normally not possible with photometry. This method is called transit timing variation, or TTV.

In this project we will examine the system of Kepler-36, to determine the TTVs of the signal, and determine different orbit parameters.

3 Kepler-36

The planetary system Kepler-36 was described in Carter et al. 2012, pointing out an exception from the orbit-composition pattern in the Solar System, with a gas-planet and a rocky planet very close to each other. The two known planets, Kepler-36b and Kepler-36c, have orbital semi-major axes differing only by 0.013 AU. Nevertheless Kepler-36b has 8 times the density of Kepler-36c, implying that they have completely different compositions; Kepler-36b is a rocky super-Earth, while Kepler-36c is a Neptune-sized gas giant. The two planets are approximately in 6:7 resonance.

From Carter et al. 2012 it is seen, that the central star is a subgiant slightly hotter and heavier than the sun, with a temperature of $T_{\text{eff}} = 5911(66) \text{ K}$, a mass of $M_\star = 1.071(43) M_\odot$ and at larger radius, $R_\star = 1.626(19) R_\odot$.

4 Theory

In this section the method of discovering transiting exoplanets will briefly be presented. This is done in order to explain and characterize TTVs.
4.1 Transits

When searching for exoplanets many different methods are being used. With the help of the Kepler satellite, transit photometry is the method which has detected the largest numbers of planets to date [Exoplanet Catalog].

When making photometry observations of a star, the amount of flux from the star is measured. When a planet moves in front of its host star, as viewed from the observer, some of the starlight will be blocked. This leads to a temporary decrease in the received flux, lasting until the planet is no longer in front of the star. This phenomenon is known as a transit, and can be seen on figure 1.

![Transit Diagram]

Figure 1: The figure shows the light curve for a transiting planet. It is from Winn 2010, figure 1.

By measuring the same star for an extended period of time, it is possible to get a time series containing several transits, depending on the orbital period of the planet. From this data, different characteristics of the planet and its host star can be deduced, such as the orbital period, the mid-transit time, the planet-star radius ratio, the inclination of the system and the semi-major axis.

In the case of a planet, which is only gravitationally affected by its host star, it is not possible to determine the mass of the planet using transit photometry. Instead, this can be done using
another method to detect exoplanets: the radial velocity method. This method makes use of the gravitational pull of the planet on the host star, revealing itself as a periodic Doppler-shift in the spectrum of the star. A major limitation in exclusively using this method is that it is only possible to determine a minimum mass of the planet, $M_p \sin(i)$, where $M_p$ is the true mass and $i$ is the inclination of the orbit. However, the inclination can be found using the transit photometry method, so by combining these, the mass of the planet can be determined using the following [Winn 2010, eq. 25]:

$$\frac{M_p \sin(i)}{(M_p + M_\star)^{2/3}} = K_\star \sqrt{1 - e^2} \left( \frac{P}{2\pi G} \right)^{1/3},$$

where $M_\star$ is the mass of the star, $K_\star$ is the semi-amplitude of the Doppler-shifted signal, $e$ is the eccentricity and $P$ is the period.

4.2 TTVs

From the Solar System it is known that a star is not confined to having only one planet in orbit. The system might as well contain several planets, thereby becoming a multiplanetary system. In this case each planet is not only gravitationally affected by the host star, but also by the other planets in the system. This leads to changes in the orbits of the planets; changes that can be seen in the time series, if the spectral resolution and the mutual gravitational pull of the planets is sufficiently large. Here the time between the mid-transit times of different transits will no longer be constant, hence the name Transit Timing Variations. This leads to non-constant periods. An illustration of the difference in mid-transit times between single- and multiplanetary systems can be seen on figure 2.

Both plots show several normalized transits of Kepler-4b (figure 2a) and Kepler-36c (figure 2b) plotted above each other. Each of these has been cut out using a constant period, and not by centering the mid-transit times around zero. This difference in cutting obviously means nothing for the single-planetary system of Kepler-4b. But for Kepler-36c it makes a tremendous difference. Here each mid-transit time is shifted compared to the first transit. This shifting is seen to be periodic.

The periodicity in the transit times is the reason why this method is so powerful. Not only does it reveal another object interfering with the motion of the star and planet; it can also be used to calculate the mass of both the planet and the other object. This can be done in several ways, depending on the characteristics of the orbits. The general method is to make an n-body simulation, where the mass of each planet is varied, until it fits the observations. However, this method is not used in this report, and will therefore not be discussed further. Instead it is used, that the system examined here consists of two planets in approximate resonance with each other. For such a system the following expression is valid [Agol, Steffen, Clarkson, et al. 2005]:

$$\delta t_{\text{max,1}} \approx \frac{P_1}{4.5j} \frac{m_2}{m_2 + m_1},$$

where $\delta t_{\text{max,1}}$ is the maximum transit timing deviation, $j$ is the order of resonance, $P_1$ is the average time between transits, $m_1$ is the mass of this planet and $m_2$ is the mass of the planet or other object that causes the perturbation of the orbit. In a first order resonance system, the $j$-value can be calculated using $P_1 \cdot j = P_2 \cdot (j + 1)$. In the case of Kepler-36 $j = 6$. 
Figure 2: The stacked transit of Kepler-4 and Kepler-36.

(a) The stacked transit intervals with normalized flux for Kepler-4, cut with a period of 3.2136641 days.

(b) The stacked transit intervals with normalized flux for Kepler-36c, cut with a period of 16.231722 days.

If the transits of both objects can be detected, equation (2) can be written for both, with only $m_1$ and $m_2$ as unknowns. By using these two equations the mass ratio of the planets can thereby be calculated.

To get an absolute value of the two masses, one needs to determine the transit duration time. In transits with no TTVs, the mass cannot be determined from the duration time, since a small planet in close orbit to its star can have the same transit duration time as a larger planet further out. But if these duration times are combined with the mass ratio, it is possible to find independent values of $m_1$ and $m_2$.

5 Data analysis

The data analysis is carried out from all of the 17 quarters of Kepler data. The Kepler satellite measures in two different time intervals: short and long cadence, with time intervals of one minute and 30 minutes respectively. In this analysis the long cadence data is used. This minimizes the noise, making the transits more distinguishable from the rest of the light curve. In return, the number of data is obviously not as high compared to short cadence, which can be a problem if the transit duration is short. The first task is to remove all the outliers from the data set. These outliers can be caused by saturation of the CCD, scattered light or other false signals.

To remove these outliers the data is first normalized by dividing with a moving median filter. For each point it takes the median with the two neighbouring points. This can be seen as the black dots on Figure 3. The standard deviation is then calculated and can be seen as the red line on figure 3. All the points more than three times the standard deviation away from the mean are then replaced
with the value from the moving median filter. The TTV's for the two different planets can then be determined from this data set without outliers.

Figure 3: The data normalized by a moving median filter. The red lines mark the cut-off limit at a distance of $3\sigma$ away from the mean.

The method for determining the TTVs will be described based on the planet Kepler-36c, which has the deepest transits of the two planets in the system. To determine the value for the mid-transit time for each transit, $T_0$, the data is cut around each transit, which makes the fitting to the model much easier. Dividing the data set into intervals is done using the period and $T_0$ value from Carter et al. 2012. The data set is then cut into intervals, with each interval given by

$$n \cdot P + T_0 \pm \Delta T,$$

where $\pm \Delta T$ is 1.25 days to make sure that the transit is inside the interval even with the largest TTV. To visualize the effect from the TTVs, all the cut intervals can be stacked on top of each other, which should give a vertical sinusoidal curve centred around zero, according to Carter et al. 2012. But if the period and $T_0$ value from the article is used as mentioned earlier, the curve looks like Figure 4, which is clearly not centred around zero. This figure shows the intervals with normalized flux; the method of this normalisation will be discussed later. To find the best period to fit all the transits, the Mandel and Agol model [Mandel and Agol 2002] is fitted to all the normalized data. The period which makes the best fit between the model and the data points is used. All the normalised intervals stacked on top of each other with this calculated period can be seen on figure 2b, which is then nicely centred around zero. This period is determined to be 16.231722(9) days in contrast to the period $P = 16.22407$ days from Carter et al. 2012.

It is clearly seen from figure 5, a cutout interval from the data set, that the raw data is not normalized. To fit the model properly to the data, using the parameters from the article, each individual data interval needs to be normalized. The normalization factor is only determined from the data points outside the transit, which is visualized by the blue crosses in figure 5. Outside the
transit is defined to be five hours away on each side from the minimum flux in each cutout. The normalization factor is given by a polynomial at the order of 1-5. The best fit between the data points and these five polynomials is determined by the BIC value. The BIC value is given by

\[
\text{BIC} = \chi^2 + k \cdot \ln N, \quad (4)
\]

where \( k \) is the order of the polynomial, \( N \) the total number of data points, and \( \chi^2 \) is determined from the least squares method, given by

\[
\chi^2 = \sum_{i=1}^{N} \left( \frac{\text{data}_i - \text{model}_i}{\text{error}_i} \right)^2. \quad (5)
\]

An example of the polynomial with the lowest BIC value is also shown in figure 5 as the red line. After determining the polynomial with the lowest BIC value, all the data points in each interval are then divided by this polynomial. The same interval, but now normalized can be seen in figure 6.

The model of Mandel and Agol 2002 is now used on each cut-out to fit \( T_0 \) of each transit separately – the remaining transit parameters are locked down using the values from Carter et al. 2012. The
Figure 5: A cutout interval of the data, displaying the 51st transit. The black dots are all the data points in the cutout, the blue crosses are the points used to calculate the normalization factor, and the red line is the final polynomial used for the normalization.

Figure 6: The cutout data interval after the normalization of the data by the BIC polynomial.

Markov chain Monte Carlo (MCMC) implementation from the PyAstronomy package\(^1\) is used in order to also obtain proper errors on the fitted values. To correct for limb-darkening the quadratic limb-darkening approximation [Sing 2010] with parameters \(a = 0.3539\) and \(b = 0.2851\) is used – also implemented in the PyAstronomy package. These values are found using the effective temperature of 5911(66) K from Carter et al. 2012.

First, the minimum of the light curve section is chosen as a first guess of \(T_0\). Then a standard fit is made for a more precise estimate of this value. The normalized light curve is divided with the fitted model and the standard deviation of this vector is used as a constant error on the flux. Then

\[^1\text{http://www.hs.uni-hamburg.de/DE/Ins/Per/Czesla/PyA/PyA/}\]
a full MCMC-fit is performed on the section with 10 000 iterations and steps of 30 minutes. The algorithm outputs a binary chain file, from which the 1σ error of the probability density distribution and $T_0$ as the median value is obtained.

For each section an epoch number starting from zero at the first transit (using the known period) is assigned. From the values of the now known $T_0$ it is possible to determine $\delta t_{\text{max}}$.

## 6 Results

When performing the data analysis we easily found the transits for Kepler-36c. This is no surprise, since according to Carter et al. 2012 this planet is a large Neptune-like planet, which made the transits deep. However, trying to detect the transits for Kepler-36b was very challenging. We did not manage to find the transits using our data analysis program, and suspect that Carter et al. 2012 used a different method in obtaining this planet’s transits. The results and analysis is therefore only based on Kepler-36c.

When the data for this planet has been reduced and normalized, and the mid-transit times has been detected, the mid-transit times can be plotted as a function of their epoch. This can be seen on figure 7. From this figure the TTVs cannot be seen. This is because the variations are very small compared to the span of the Kepler data. Therefore the data is divided by $(\text{epoch} \cdot P + T_{0,0})$, where $P$ and $T_{0,0}$ are the values determined above. This would be the mid-transit time at each epoch if there was no TTVs. By subtracting this relation from the data in figure 7, the TTVs becomes visible. This can be seen on figure 8.

The data in this plot is seen to be drifting at the end. This can be due to the period being slightly wrong, or more exciting that there may be a fourth object in the system, besides the star and the two planets. This could be another planet or maybe a moon. Further research needs to be done to determine the cause of this drift.

By using the data from Carter et al. 2012 combined with equation (2), the values for $\delta t_{\text{max}}$ according to the paper can be calculated for both planets. These values divided by two are seen on figure 9 as the red and green lines for Kepler-36b and Kepler-36c respectively. The blue line in this figure is following the trend of the data, and it can be used to guide the eye. The values of $\delta t_{\text{max}}$ for Kepler-36c, calculated from Carter et al. 2012, is seen to agree well with the results obtained in this project. We suspect therefore that the value of $\delta t_{\text{max}}$ for Kepler-36b would also fit the data well, if the transits of this could be detected.
Figure 8: The mid transit time subtracted by \((\text{epoch} \cdot P + T_{0,0})\), the line from 7, as a function of the epoch.

Since we do not have a direct measurement of \(\delta t_{\text{max}}\) for Kepler-36b, we were not able to calculate the individual masses of the two planets. But using \(\delta t_{\text{max}}\) and the period of Kepler-36c, we were able to get the relation between the masses from equation (2). During this we get \(\frac{m_b}{m_b + m_c} = 0.3\). In Carter et al. 2012 the individual masses were found to be \(4.45^{+0.33}_{-0.27}M_\oplus\) and \(8.08^{+0.60}_{-0.46}M_\oplus\) for Kepler-36b and Kepler-36c respectively. For these masses the mass-relation becomes \(\frac{m_b}{m_b + m_c} = 0.36\), which is close to our result.

If these masses were to be determined using data from the radial velocity method, we calculate from (1) that the semi-amplitude needs to be \(K_b = 1\) m/s and \(K_c = 2\) m/s. This is a very small signal, and can only be detected from very high resolution spectrographs for very bright stars. Since \(V = 12\) for this star, the masses would be very hard to detect using the radial velocity method. This emphasizes the strength of the TTV-method.

7 Conclusion

In this report the system of Kepler-36 has been examined. It was only possible to detect transits for Kepler-36c, which was the heavier of the two planets. Therefore parameters were determined for this planet only. From the transit intervals we discovered that the average period reported in Carter et al. 2012 did not fit the data. We therefore found a period which fitted the data better.

From the period and the transit times found from the fit, it was possible, to determine the relation between the masses of the two planets. This result fits well with the results from Carter et al. 2012. Using data from this article, we found that the planets in the system would be very hard to detect using the radial velocity method.

From this project we have both seen the weakness and strength of using TTVs: it is hard to detect the transits, because of the varying transit times, but when the transits are detected, they give more information about the system compared to 'regular' transit photometry.

If only one planet is discovered in a planetary system, and the transit time is seen to vary, one can deduce that there must be another body in the system. This can be either another planet, another star or an exomoon. It is believed that TTVs could become the first method to discover
Figure 9: The final O-C diagram for Kepler-36c. The yellow points shows the mid transit time minus the calculated line from 7 with errorbars, as a function of epoch. The red and green line are the calculated value of $\delta t_{max,c}$ for Kepler-36b and -c respectively, and the blue curve is a sinusoidal guideline for the eye.

an exomoon. In the future, with better spectral resolution, this method may also be used in radial velocity measurements.
References


